

Magnetics Design for Power Electronics

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Introduction

The goal of this document is to cover the fundamental formulas and concepts that allow an engineer to analyze and design magnetic components for power electronics. Application examples will focus on extreme design requirements, such as aircraft avionics, but the principles are applicable to any requirements. Design examples will focus on applications where off the shelf components are unavailable or inadequate. While theory is critical to understanding magnetic components and getting a design started, several critical factors simply *must* be tested before the design can be finalized.

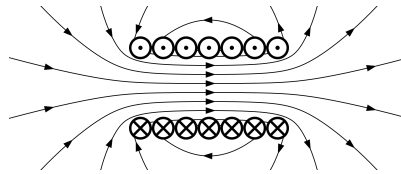
All formulas in this text use SI units (Sorry Jeff Brewer...)

Chapter 1

Fundamental Formulas and Units

This section will cover magnetic theory from a physics standpoint and introduce the relevant variable names, units, and some fundamental formulas. Links are provided for more thorough theoretical discussions.

1.1 Magnetic Fields



Magnetic fields are classically drawn as a set of curved lines flowing into and out of some sort of magnet. These lines represent the two different magnetic fields at the same time, \mathbf{B} and \mathbf{H} . Technically, the term "magnetic field" is ambiguous and does not refer to \mathbf{B} or \mathbf{H} specifically.

The \mathbf{B} field is measured in the SI unit of Teslas" (T), or Gauss in imperial units. 10,000 Gauss = 1T. A common and descriptive name for the \mathbf{B} field is "magnetic flux density". It is used in physics to calculate the force on a charged particle that is moving through a magnetic field. For power electronics, the time-varying \mathbf{B} field combined with the physical parameters of the magnetic component is related voltages across its windings.

The \mathbf{H} field is measured in SI units of Amps/meter (A/m), or Oersteds (Oe) in imperial units. $1 \text{ A/m} = \frac{4\pi}{1000}$ Oersteds. It is often referred to as "magnetic field strength". It is less commonly used in physics, but in power electronics the \mathbf{H} field combined with the physical parameters of the magnetic component is related to the current flowing through its windings.

The strength and direction the \mathbf{B} and \mathbf{H} fields are described as a [vector fields](#). While the \mathbf{B} and \mathbf{H} fields are different, they are closely related. At the basic level the two fields are aligned, so both \mathbf{B} and \mathbf{H} vector fields point in the same direction at any given point. The relative \mathbf{B} and \mathbf{H} vector field strengths are also described by the formula:

$$\mathbf{B} = \mu\mathbf{H}$$

Where μ is the [permeability](#) of the material where the fields exist.

Magnetic fields are also governed by [Maxwell's Equations](#), specifically [Gauss's law for magnetism](#), which states that $\nabla \cdot \mathbf{B} = 0$, or the divergence of the magnetic field is always zero. What this means practically is that any magnet or electromagnet cannot create a net magnetic field, as all magnetic field lines that "exit" one pole must "return" to its other pole. This concept is similar to Kirchhoff's current law, where all electric current that flows out of a source must eventually return to the source's other terminal.

1.2 Permeability

[Permeability](#) is a material property. It is expressed as a scalar value, μ , times the permeability of free space, $\mu_0 = 4\pi 10^{-7} \text{ H m}^{-1}$. All materials (except [diamagnetics](#)) have a permeability greater than or equal to μ_0 . Air and

other nonmagnetic materials (like FR4 and copper) have a permeability of μ_0 . For power electronics, materials with permeabilities of 10μ to $100,000\mu$ are used.

There notation for permeability can be confusing. All magnetics equations require the use of absolute permeability, and often simply use the variable μ . But the permeability of magnetics materials is always expressed as scalar multiple of the permeability of free space, but also uses the same symbol, μ . For example, a ferrite core with a permeability 3,000 times more than μ_0 will show "3000 μ " on its material datasheet. This is an expression of relative permeability. The absolute permeability would be $\mu = 3000\mu_0 = 3.77 \times 10^{-3} \text{ H m}^{-1}$.

The formulas in this paper will stick to the industry standard notation, despite the ambiguity. As a sanity check, no material available for power electronics will result in an absolute permeability μ in an equation being greater than $\mu \leq 1.26 \text{ H m}^{-1}$, and in most cases will be less than or equal to $\mu \leq 0.025 \text{ H m}^{-1}$.

At high frequencies the permeability of materials is not perfectly scalar, but is rather a complex number. As the imaginary component of the permeability increases, the \mathbf{B} and \mathbf{H} fields lose their alignment. The effects of complex permeability can be safely ignored for practical power magnetics design.

1.3 Magnetic Field Energy

The energy stored per unit volume (volumetric energy density) in a magnetic field is:

$$U = \frac{\mathbf{B} \cdot \mathbf{H}}{2}, \text{ J/m}^3$$

The total energy stored is equal to the energy density times the volume in which the field is present

$$E = V_e U = V_e \frac{\mathbf{B} \cdot \mathbf{H}}{2}, \text{ J}$$

Using the $\mathbf{B} = \mu\mathbf{H}$ formula, the magnetic field energy can also be computed as:

$$E = V_e \frac{\mathbf{B} \cdot \mathbf{H}}{2} = V_e \frac{\mathbf{B}^2}{2\mu} = V_e \frac{\mu\mathbf{H}^2}{2}, \text{ J}$$

An important takeaway from these formulas is that for a given flux density, low-permeability materials store more energy per unit volume. Another way of viewing this phenomenon is that it is harder, and therefore takes more energy to create the same magnetic field in a low permeability material vs a highly permeable one.

1.4 Physical Properties of a Magnetic Core

There are five critical dimensional properties of a magnetic core used in design calculations:

1. **Effective Core Area A_e , m^2**

The cross-sectional area of the core, measured as a plane that is perpendicular to the magnetic path. Some manufacturers specify an $A_{e_{min}}$ as well. This is the smallest core area cross section along the entire magnetic path. For conservative saturation calculations, use $A_{e_{min}}$. For core loss calculations use A_e .

2. **Effective Magnetic Path Length L_e , m**

Sometimes used when doing calculations for gapped inductors.

3. **Inductance per Turn², A_L , H/Turn²**

This parameter is almost always given for toroidal cores when the material is also specified. It can be calculated from A_e , L_e , and μ . This value is reduced by the addition of an air gap. The industry standard units are nH/Turn².

4. **Winding Window Dimensions**

The window dimensions combined with the required number of turns determine the DC and AC resistance of a coil. DC resistance is always reduced with a larger window area, but AC resistance is more complicated. In general, a wider window area will allow more turns per layer, reducing AC resistance. When a winding bobbin is used, the bobbin determines the window dimensions.

5. **Effective Core Volume V_e m^3**

Used only for core loss calculations, but also contributes to component weight, as most magnetic materials are dense.

Magnetic cores are available in a variety of shapes and sizes. From a theoretical standpoint, a core of any shape can be accurately described and analyzed in terms of the above parameters, so long as the effective core area is nearly constant around the magnetic path length. However, there are often practical trade offs between different core sizes such as easy of mounting, assembly, versatility, and shielding.

The core material is also critical. There are several important core properties that are all determined by the core material. Unlike dimensional properties, magnetic material properties are heavily dependent on operating and environmental variables.

1. Saturation Flux Density (vs temperature)

Unlike nonmagnetic materials, magnetic materials lose effectiveness as the flux density increases. At some point, the core is considered "saturated" and its permeability decreases. Some materials, like silicon steel, have very sharp saturation characteristics. Others, like powder cores, saturate softly. Avoiding core saturation is often a key design constraint.

2. Permeability (vs temperature, flux density (B), and frequency)

For switching applications, the exact permeability is generally unimportant. Power ferrite materials generally have usable permeability over standard temperature, flux density, and switching frequency ranges. However, for high-permeability materials used specifically for common mode-chokes and other AC only filters, it is often very hard to find a material that maintains high permeability over the required frequency and temperature range.

3. Core Loss (vs temperature, half the flux density swing ($\Delta B/2$), and frequency)

Core loss is often given as a formula that calculates loss per unit volume. The formula is a curve fit based on Steinmetz's equation $P_{core} = k \cdot f^a \cdot (\frac{\Delta B}{2})^b$ where k , a , and b are coefficients determined by empirical testing. The coefficient a is always between 1.0 and 2.0, and b is always between 2.0 and 3.0. The exponential nature of the formula means that core losses increase very rapidly with increasing frequency and flux density swing. The coefficients are also determined by testing with sine waves, so the losses in a square wave converter are often worse due to high frequency harmonics.

Also, when given a set of core loss curves and a set of Steinmetz coefficients, the curves are more accurate. A single set of Steinmetz coefficients cannot accurately describe a full set of loss curves, but is instead a curve fit around a pre-selected operating point. The Steinmetz coefficients may be inaccurate if the design's operating point is different than the curve-fit point.

4. B – H Loop (vs temperature)

The B – H Loop can be used to show how sharp the onset of core saturation is. It can also be a useful tool for visualizing the various operating points of a magnetic component.

There are four general material categories used for magnetic components

1. Metal laminated cores

Metal laminated cores consist of a thin metal that is either stamped into E I shapes and stacked, or a thin ribbon that is wound together. These materials have very high permeability ($5,000\mu$ - $100,000\mu$), and very high saturation flux densities of 1.0T - 2.4T. Their saturation characteristics are usually sharp. However, many of these materials perform poorly or have high losses at high switching frequencies, making them unsuitable for most avionics switching converters. These materials are better suited for 400Hz AC and DC filtering applications.

2. Ferrites

Ferrites are pressed/sintered powder cores. These materials typically have high permeability ($2,000\mu$ - $20,000\mu$), and have saturation flux densities of 0.2T - 0.4T. Their saturation characteristics are usually sharp. Ferrites offer good high frequency performance (50kHz - 2Mhz) and the lowest core losses for most switching applications. They are the default material for high frequency power transformers. Ferrite cores must be gapped for power inductor applications.

3. Powder Cores

Powder cores are also pressed/sintered cores, but the magnetic particles are not as closely bonded, resulting in lower permeability materials (14μ - 550μ). Each material is offered in discrete permeability values. They have medium saturation flux densities of 0.8T - 1.5T, but due to their extremely soft saturation characteristics they begin to lose permeability well below their saturation point. Their low permeability makes them unsuitable

for most transformer applications, but allows them to be used for power inductor applications without an air gap. Powder cores have manageable core losses for medium/low switching frequencies (25kHz - 100kHz). They are typically used as toroids, which are simple and robust for low turn count inductors, but must be potted to secure the windings. High turn counts are time-consuming to wind, and it is not easy to wrap insulating tape between layers for high isolation voltage coupled inductor applications.

4. Nanocrystalline Cores

Nanocrystalline cores are typically tape-wound toroids that have excellent properties for use in common mode chokes. They have extremely high initial permeability (20,000 μ - 100,000 μ), and high saturation flux density. These features make them very useful for kHz range common mode filtering. They also hold usable permeability into and beyond MHz frequencies, out-performing high permeability ferrites. They are also relatively stable vs temperature, giving them a large advantage over ferrites for low temperature operation. Nanocrystalline materials have high core losses once frequency is above the permeability roll-off point. For filtering applications, this is beneficial, since it provides valuable damping properties. The core loss acts as a series resistance, but only for high frequencies. This can avoid the need for common-mode RC damping networks. Overall, nanocrystalline cores offer the best performance for common mode chokes, but are more expensive due to the material cost, and the complexity of winding toroids.

5. Air Cores

Despite the wide variety of magnetic materials available, air is an essential material for magnetics design. Its permeability is as low as it gets, but that also means it stores the most energy per unit volume. It is also stable over temperature, never saturates, is essentially lossless, and usable at any frequency. No losses and high usable frequency makes air indispensable for high Q RF inductors. It is rare that engineers get to work with "ideal" materials, but air is an essential and ideal material for use in magnetics design. That is a great thing, because *everything else* in magnetics design is far from ideal!

1.5 B field units

There are [several ways](#) to break down the **B** field units, teslas, into other SI units. The one most relevant to electronics is:

$$\mathbf{B} = \text{Vs}/\text{m}^2$$

This is *not* a formula, but instead is an expression of units where V = Volts, s = seconds, and m = meters. The actual time-varying formula for flux density is

$$\mathbf{B}(t) = \frac{\int V(t)dt}{A_e}$$

where there is some single-turn loop of wire with an applied time varying voltage $V(t)$, and the A_e term is the area enclosed by the loop. This formula can be rearranged to describe the voltage in terms of **B**

$$V(t) = \frac{d\mathbf{B}(t)}{dt} A_e$$

For a coil of wire wound around a magnetic core, the formula is adjusted by a factor of the number of turns, N. Applying a voltage to a coil of several turns effectively divides the applied voltage by the number of turns. 1V across 10 turns is the same as 0.1V per turn, so the turns factor, N, goes in the denominator, decreasing the **B** field as turns increase. Additionally, it is assumed that the resultant magnetic field is contained by the core, so the m^2 area term can be replaced by the effective core area. The above formula also includes an integral. Technically that integral starts at $t = 0$, or the instant the coil exists. In power electronics design, the integral can often be evaluated over a single switching cycle to see the change in flux density throughout the switching cycle. For hard-switched converters where the voltage applied to inductors and transformers is a square wave, the integral is further simplified to the product of the applied voltage and the time it is applied for.

$$\Delta\mathbf{B} = \frac{\int_{t_0}^{t_1} V(t)dt}{NA_e}$$

$$\Delta\mathbf{B} = \frac{V \cdot s}{NA_e}$$

This is the most important formula regarding flux density for power magnetics. It is important to know when the $V \cdot s$ term can be evaluated as a simple volt-second product, or when the integral must be evaluated. The Δ in front of \mathbf{B} is intended to show that this formula cannot provide the absolute value of \mathbf{B} , but only describes how \mathbf{B} changes from its initial value as a result of applied voltage, turns, and core area.

The first few of many magnetics design trade-offs can be seen in this formula. As noted in the magnetic material section, core saturation often must be avoided, and core loss is strongly related to $\Delta\mathbf{B}$. This formula shows that to reduce $\Delta\mathbf{B}$, the applied voltage must be reduced, voltage must be applied for less time, the core area must increase, or the number of turns must increase.

This formula is also the heart of how voltage transformers work. Placing two windings on a single core forces the same flux density through them, locking their voltages to the ratio of turns.

$$V_{pri}(t) = \frac{d\mathbf{B}(t)}{dt} A_e N_{pri}, \quad V_{sec}(t) = \frac{d\mathbf{B}(t)}{dt} A_e N_{sec}$$

$$\frac{V_{pri}(t)}{N_{pri}} = \frac{d\mathbf{B}(t)}{dt} A_e = \frac{V_{sec}(t)}{N_{sec}}$$

$$\frac{V_{pri}(t)}{N_{pri}} = \frac{V_{sec}(t)}{N_{sec}}$$

1.6 H field units

The \mathbf{H} field units are:

$$\mathbf{H} = A/m$$

Where A = amperes of current flowing through a loop of wire, and m = meters of distance for the effective magnetic path length.

The time-varying $\mathbf{H}(t)$ field is based on the current flowing through a loop of wire, and the effective path length that the magnetic flux takes. The magnetic path for a coil in free-space is hard to imagine because the resulting magnetic field technically extends to infinity with varying strength. However, for a coil of wire on a magnetic core, the magnetic field is effectively contained by the core. This makes the path length much easier to visualize, and is specified by the core datasheet. This formula is also scaled by the number of turns in the coil. Because the same current is forced through each turn, the turns factor, N, goes in the numerator, increasing the \mathbf{H} field as turns increase. 1A through 2 turns is the same as 2A through 1 turn.

$$\mathbf{H}(t) = \frac{NI(t)}{L_e}$$

This formula has no integrals or derivatives, unlike the formula for flux density. This makes it particularly useful for understanding the DC operating point of power inductors.

This formula explains how currents flow in a transformer. Placing two windings on a single core forces the same \mathbf{B} field and \mathbf{H} field through them. However, one winding of a transformer is fixed to an AC voltage source, which also fixes the \mathbf{B} field. Additionally, the \mathbf{B} field and \mathbf{H} fields are related by the formula $\mathbf{B} = \mu\mathbf{H}$. For an ideal transformer with a very high permeability core, this means the \mathbf{H} field must remain very small, almost zero. If the \mathbf{H} field must remain small, and the $\mathbf{H} = NI(t)/L_e$, how can transformers carry any current? Just how multiple current carrying turns add together to increase the \mathbf{H} field, turns carrying current in opposite directions can cancel out.

$$\mathbf{H}(t) \approx 0 \approx \frac{N_{pri}I_{pri}(t)}{L_e} + \frac{N_{sec}I_{sec}(t)}{L_e}$$

$$0 \approx N_{pri}I_{pri}(t) + N_{sec}I_{sec}(t)$$

$$N_{pri}I_{pri}(t) = -N_{sec}I_{sec}(t)$$

The negative sign on the secondary side indicates that while current flows into the primary, current flows out of the secondary.

1.7 How B and H Relate to Inductance

While the basic formulas for **B** and **H** relate directly to how transformers work, a bit more effort is required to see how they relate to inductance. The SI unit of inductance is the Henry. It can be expressed as [several different combinations](#) of other SI units, but the one most relevant to power electronics is:

$$H = \frac{V \cdot s}{A}$$

Where V = volts, s = seconds, and A = amperes. By expanding on the units of the **B** and **H** fields and including the turns factor for each, we can re-arrange the formula $\mathbf{B} = \mu\mathbf{H}$ to express inductance as a function of turns and magnetic core properties.

$$\begin{aligned}\mathbf{B} &= \mu\mathbf{H} \\ \frac{V \cdot s}{NA_e} &= \mu \frac{NA}{L_m} \\ \frac{V \cdot s}{A} &= \mu \frac{N^2 A_e}{L_m} \\ L &= \mu \frac{N^2 A_e}{L_m}, \text{ H}\end{aligned}$$

This formula demonstrates some fundamental trade-offs of power inductor design. To increase inductance, turns must be increased, the core area must be increased, the core permeability must be increased, or the magnetic path length must be shortened.

The A_L inductance value given on many core datasheets is simply the value of $\mu \frac{A_e}{L_m}$ for a given core shape, size, and material. This A_L value multiplied by N^2 gives the inductance of the winding in Henries.

$$\begin{aligned}A_L &= \mu \frac{A_e}{L_m}, \text{ H/N}^2 \\ L &= N^2 A_L, \text{ H}\end{aligned}$$

1.8 Magnetic Circuits, Flux, and Reluctance

The previous sections discuss the formulas required to analyze a magnetic component with a core made of a material with a consistent permeability. For toroidal transformers, or inductors made with powder cores, no further formulas are necessary. However, at some point air gaps in inductors must be addressed, but before that can be done a new method of magnetic analysis must be discussed.

The motivation behind analyzing a magnetic component with a circuit-like analogy is to provide clear separation between the physical core constants (A_e , L_e , and μ), the winding constants (N for each winding present) and the instantaneous current for each winding, and the instantaneous magnetic flux in the core.

Note that the term used above was magnetic *flux*. Not magnetic *flux density*. The **B** field, magnetic flux density, has units $\frac{V \cdot s}{NA_e}$. Note the area term in the denominator. Flux density is simply units of *flux* per unit area, so flux units of $\frac{V \cdot s}{N}$, and is represented by Φ .

After all that, on to the magnetic circuit analogy:

- 1 Every winding on a core acts like a voltage source. Instead of applying a voltage, it applies a Magnetomotive Force (MMF) equal to the instantaneous current multiplied by the number of turns. $MMF = NI(t)$
- 2 Windings on the same core act like they are connected in series. Just like voltage sources, if their currents have the same polarity, their MMF's add together. If the currents have opposite polarity, their MMF's subtract
- 3 Flux (Φ) acts like an electric current. It flows out of an MMF source, through some conductive path, and returns to the source's other terminal. No flux is created or lost, it only flows.
- 4 For any toroidal core or basic U or E core, the same flux flows through both coils. (In reality, there is some leakage flux that flows through the air and not the core, but it is negligible for now)

- 5 All the core's physical properties are lumped into a passive element that acts like a resistor. Instead of having *resistance* R , it has *reluctance* \mathcal{R} . It is passive and linear.**
- 6 Ohm's law follows the formula $V(t) = I(t)R$. The magnetic circuit equivalent is $MMF(t) = \Phi(t)\mathcal{R}$. This assumes that all series voltage/MMF sources have been summed into a single applied voltage/MMF.**

To use magnetic circuit analysis, we must derive the formula for reluctance \mathcal{R} . Re arranging the magnetic circuit formula, we get $\mathcal{R} = \frac{NI(t)}{\Phi(t)}$. From that formula and the rules of the circuit analogy above, we know that reluctance contains all of the units that are not amps, flux, or turns. We also know that the units of flux are $\Phi = \frac{V \cdot s}{N}$.

$$\begin{aligned} \mathbf{B} &= \mu \mathbf{H} \\ \frac{V \cdot s}{NA_e} &= \mu \frac{NA}{L_m} \\ \frac{L_m}{\mu A_e} &= \frac{NA}{\frac{V \cdot s}{N}} = \frac{NA}{\Phi} \\ \mathcal{R} &= \frac{L_m}{\mu A_e} \end{aligned}$$

This formula relates all three physical core constants into an effective reluctance \mathcal{R} . Also note, that this term is simply $1/A_L$, so the inductance of a magnetic component can also be defined by turns and reluctance.

$$\begin{aligned} \mathcal{R} &= \frac{L_m}{\mu A_e}, \quad A_L = \mu \frac{A_e}{L_m} \\ A_L &= \frac{1}{\mathcal{R}} \\ L &= \frac{N^2}{\mathcal{R}} \end{aligned}$$

Now that reluctance is easily calculated, the concept can be applied to analysis of air gaps in inductors. Ohm's law shows that resistances in series can simply be added together. Does Ohm's law care what material the resistor is made of? No. Anything chunk of material that conducts a DC current when a DC voltage is applied is a resistor in one way or another. In the same manner, anything that has flux flowing through it in a magnetic circuit has a finite reluctance. Except for magnetic in circuits, materials like air, plastics, and FR4 are not insulators! Any common material that isn't magnetic has a permeability of at least μ_0 , and therefore can "conduct" magnetic flux in the presence of a MMF. This means if we stick a gap of air somewhere in a flux path of magnetic device's core, if we can calculate the reluctance of the air gap, we can simply add it in series with the reluctance of the core. This idea lets us add more rules to the magnetic circuit analogy:

- 7 Reluctance can be calculated for any material, where $\mathcal{R} = \frac{L_m}{\mu A_e}$. For an air gap, $\mu = \mu_0 = 4\pi 10^{-6} \text{ H m}^{-1}$, and $L_m = L_g$, the gap length in meters. The effective gap area is equal to the core area A_e (For gaps where L_g is much smaller than the diameter of a round core cross-section, or the short side of a rectangular core cross-section).**
- 8 Reluctances in series can be quantified and added together to form the total effective reluctance of a core.**

1.9 Effects and uses of Air Gaps

1.9.1 Air Gaps in Power Inductors

With an understanding of magnetic circuits and reluctance, magnet components consisting of a high permeability core and one or more air gaps can be analyzed. How is this useful? After all, magnetic materials are specifically designed to have high permeability, and therefore very low reluctance. Why would adding an air gap, which has a low permeability and high reluctance, help?

In a general sense, the reason why inductors are used in power electronics is they store energy. Going back to one of the equations for the energy stored in a magnetic field:

$$E = V_e \frac{B^2}{2\mu}$$

Note that the energy stored is inversely related to permeability. For a typical ferrite magnetic core with a permeability of $3,000\mu$, an air gap will store 3,000 times as much energy per unit volume than the actual ferrite core! So an air gap with length 1/3000th of L_e would store just as much energy as the core itself at any given flux density. And since we know that all high-permeability core materials will saturate with a high enough flux density, or will suffer from higher core losses, adding an air gap is a straightforward way to increase the energy storage capability of a power inductor without increasing the flux density.

One fundamental constraint of power inductor design is the core must not saturate. Using magnetic circuit analysis, we know:

$$\begin{aligned}\Phi_{sat} &= B_{sat}A_e \\ \frac{NI_{peak}}{\mathcal{R}} &\leq \Phi_{sat} \\ \frac{NI_{peak}}{\mathcal{R}} &\leq B_{sat}A_e\end{aligned}$$

This formula demonstrates a trade-off between turns and reluctance. A_e and B_{sat} are considered constants for a given core size and material. I_{peak} is considered a constant derived from design requirements. That leaves two variables, \mathcal{R} and N . To solve for a required reluctance, another formula is needed. When designing a power inductor, just as the peak current handling capability must be greater than I_{peak} , the inductor must also satisfy some minimum inductance value L_{min} .

$$\frac{N^2}{\mathcal{R}} \geq L_{min}$$

By solving this equation for turns and substituting it into the equation with flux density, a formula for a minimum reluctance value is obtained.

$$\begin{aligned}N &\geq \sqrt{L_{min}\mathcal{R}} \\ \frac{I_{peak}\sqrt{L_{min}\mathcal{R}}}{\mathcal{R}} &\leq B_{sat}A_e \\ \sqrt{L_{min}\mathcal{R}} &\leq \frac{\mathcal{R}B_{sat}A_e}{I_{peak}} \\ L_{min}\mathcal{R} &\leq \frac{\mathcal{R}^2B_{sat}^2A_e^2}{I_{peak}^2} \\ \mathcal{R} &\geq \frac{L_{min}I_{peak}^2}{B_{sat}^2A_e^2}\end{aligned}$$

With minimum Reluctance value calculated, the minimum number of turns can also be obtained:

$$\begin{aligned}\frac{N^2}{\mathcal{R}} &\geq L_{min} \\ N &\geq \sqrt{L_{min}\mathcal{R}}\end{aligned}$$

These two design constraints are useful for applications where the peak current carrying capability and inductance is known, such as for three-phase 400Hz rectifier filters, and other DC filter inductors. Other applications, like switching power supply inductors, L_{min} is often not strictly known, and therefore I_{peak} is also not known.

Understanding that a key design constraint hinges on having some *minimum* core reluctance demonstrates the purpose of air gaps. They allow an inductor built with a given core to store more energy, but at the expense of needing more turns.

1.9.2 Air Gaps in Transformers

So far the analysis of magnetic circuits has been focused on inductors. Are air gaps useful for transformers? The short answer is no, air gaps are not useful for most transformers. (Except for flyback "transformers", which are really coupled inductors).

The magnetic circuit for a two-winding transformer has two MMF sources in series, and the circuit loop is completed by the core reluctance. For a transformer, the currents for each winding flow in opposite directions, so their MMF's cancel out. In reality, they cannot cancel out completely, because then no flux would flow. Flux must flow through the windings according to:

$$V(t) = \frac{d\mathbf{B}(t)}{dt} A_e$$
$$V(t) = \frac{d\Phi(t)}{dt}$$

So for any real transformer, there must be a slight imbalance between the MMF's of each winding so that some flux can flow. And given that $\Phi = \frac{\text{MMF}}{\mathcal{R}}$, in order to minimize the imbalance between primary and secondary MMF's (and therefore currents), minimizing \mathcal{R} is desirable.

This aligns with the classic analysis of a real transformer as an ideal transformer with an inductor L_m in parallel with the primary. That inductor is called the "magnetizing inductance", and accounts for the difference between the real primary current and the ideal primary current. Using magnetic circuit analysis, the magnetizing inductance can be calculated as:

$$L_m = \frac{N_{pri}^2}{\mathcal{R}}$$

Adding an air gap would only increase the reluctance, and therefore also lower the primary inductance.

There are some specific scenarios where a transformer air gap is necessary, although they are not common most switching power supplies or sine wave AC transformers. (Keep in mind that a flyback "transformer" is actually a coupled inductor that requires energy storage in the core to operate). In standard applications, the voltages and currents of all transformer windings have no DC component. In this case, the MMF's of each winding are AC functions, and can cancel out. But if for some reason one of the transformer windings must tolerate a DC bias current, then an air gap is necessary. (One place this occurs is in single-ended class-A vacuum tube power amplifiers). Since a transformer cannot pass a DC current from primary to secondary, This leads to a DC offset in the MMF of one winding, which will not be naturally cancelled out by the other windings. The offset could be nulled with external circuitry applying a proportional DC offset to another winding (like in a [mag amp](#)), but if it is not cancelled out, the MMF offset will cause a very large flux to flow through a core with low core reluctance, causing the core to saturate. Adding an air gap increases the reluctance and keeps the core flux to manageable levels, at the expense of lowered primary inductance or increased turns.

1.9.3 Air Gap Limitations

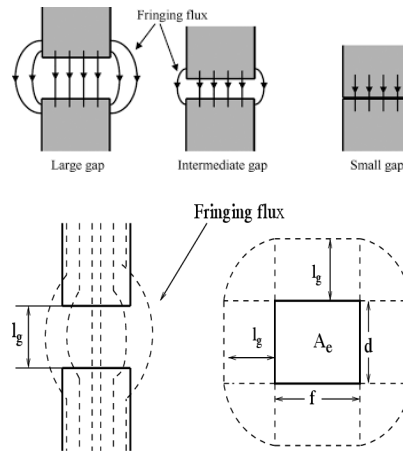
There are two main limitations to how large the air gap length, l_g , can be made for a power inductor.

- 1 As l_g increases, the required turns (for a given inductance) increases proportionally to $\sqrt{l_g}$. Therefore the DC resistance increases proportionally to l_g .**
- 2 As l_g increases, its effective area increases due to fringing flux, limiting effectiveness and potentially increasing AC losses.**

The effect on increasing turns increases the DC resistance of the winding, since a smaller wire must be used to fit the required turns in the window area, and the total length of the wire is increased due to more turns. AC resistance may change at a different rate, and [Dowells equations](#) must be used to determine the AC resistance. The upper limit of DC and AC resistance is only limited by the temperature rise of the component, which *must* be verified by testing.

The second effect of an increasing gap is fringing flux. Inside the magnetic core, the material has high permeability, and all of the flux is effectively contained. The flux is simply taking the path of least resistance, and the high permeability core is great for guiding the flux throughout its shape. However, when the flux encounters an air gap, the permeability of the gap is low, so the flux begins to spread out over a larger area.

For gaps that are very small, the linear dimensions of the core cross section are much larger than the gap length, so the flux lines stay effectively straight. The effective area of the gap is still approximately equal to the core area.



However, for larger air gaps, the fringing flux spreads farther outwards. The distance it spreads outward (with significant intensity) is approximately equal to the gap length. The first effect this has is the gap's reluctance is no longer equal to $\mathcal{R} = l_g / \mu_0 A_{e \text{ core}}$. The gap's effective area is larger than the core area, reducing the effective reluctance. To compensate, the gap length will have to be made even larger. Some core manufacturers provide a curve fit formula for core reluctance vs gap length, but practically it is often determined by testing.

Also notice how the flux begins to change direction starting inside of the core. For ferrite cores, this is negligible, as ferrites are homogeneous and amorphous, so the direction of flux does not affect its properties. However for cores using laminated metal cores, those materials are often "grain oriented", meaning they have optimized characteristics when the flux lines flow with the grain orientation. If the flux changes direction near the air gap, the core losses can increase significantly for that area.

Additionally, if the fringing flux strays far enough from the core, it can flow through the component's copper windings. If there is any significant AC flux, especially at high frequency, then the rapidly changing flux induces eddy currents inside of the copper wires, leading to heat dissipation due to resistive losses. If left unchecked, this effect can melt copper winding insulation and lead to catastrophic failure.

To mitigate the effects of fringing flux, the gap can be split up into multiple smaller gaps with the same equivalent reluctance. Most gapped cores consist of two core halves, so the gap is often split into two series gaps. Some tape-wound metal cores are cut several times to keep gap lengths manageable. If the core is not grain-oriented, or the additional core losses are tolerable, the copper windings can be physically moved away from the gap to avoid eddy current losses.

Ultimately the largest usable gap for a given core can only be determined by testing, but as a rule of thumb, if the fringing flux does not reach the windings, and the core material is ferrite, then fringing flux can be tolerated.

1.9.4 Powder Core Reluctance

So far magnetic circuit and air gap analysis has been limited to ferrite and metal cores. These concepts all apply to powder core materials. However it's rare to find a powder core that uses an air gap, because powder cores have the air gap *built in*.

Powder cores contain microscopic air gaps distributed throughout the core material, effectively reducing the permeability of the material, but in a homogeneous way instead of with discrete air gaps. This has the advantage of avoiding all fringing flux issues, and it causes all powder cores to exhibit a soft saturation characteristic. Air gaps for ferrite and metal cores assume the gap length is precise and constant across the whole core cross section. However, in powder cores, the gaps are microscopic and not precisely controlled, so some parts of the material saturate before others, softening the onset of material saturation.

Each powder core material is often offered in several different material permeability options. For example, MPP powder cores are offered in 14μ , 26μ , 60μ , 125μ , 160μ , 200μ , 300μ , and 550μ options. For powder cores, the reluctance of the core is determined by its shape (L_m and A_e) and its permeability, and is equal to $\mathcal{R} = L_e / \mu A_e$.

The same constraint formulas for minimum reluctance can be used for powder cores, except instead of choosing the air gap length, the material permeability is selected to provide an appropriate reluctance. However, due to the soft saturation characteristics of powder cores, a fixed B_{max} is typically not easily selected, and the material permeability decreases as the \mathbf{H} field (or MMF) increases. This makes the design process for powder cores slightly more complicated.

1.10 Summary of Formulas

Permeability of free space: $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{B}(t) = \frac{\int V(t) dt}{A_e}$$

$$V(t) = \frac{d\mathbf{B}(t)}{dt} A_e$$

$$\Delta \mathbf{B} = \frac{\int_{t_0}^{t_1} V(t) dt}{N A_e}$$

$$\Delta \mathbf{B} = \frac{V \cdot s}{N A_e}$$

$$\mathbf{H}(t) = \frac{NI(t)}{L_e}$$

$$U = \frac{\mathbf{B} \cdot \mathbf{H}}{2}, \text{ J/m}^3$$

$$E = V_e \frac{\mathbf{B} \cdot \mathbf{H}}{2} = V_e \frac{\mathbf{B}^2}{2\mu} = V_e \frac{\mu \mathbf{H}^2}{2}, \text{ J}$$

$$L = \mu \frac{N^2 A_e}{L_m}, \text{ H}$$

$$A_L = \mu \frac{A_e}{L_m}, \text{ H/N}^2$$

$$L = N^2 A_L, \text{ H}$$

$$\mathcal{R} = \frac{L_m}{\mu A_e}$$

$$NI(t) = \Phi(t) \mathcal{R}$$

$$L = \frac{N^2}{\mathcal{R}}$$

$$\mathcal{R} \geq \frac{L_{min} I_{peak}^2}{B_{sat}^2 A_e^2}$$

$$N \geq \sqrt{L_{min} \mathcal{R}}$$

Chapter 2

Design Examples

2.1 DC Filter Inductor

Design Requirements

- Minimum inductance at rated DC current: $L \geq 250 \times 10^{-6} \text{ H}$
- DC Current: $I_{DC} = 2 \text{ A}$
- Peak Current: $I_{peak} = 2.5 \text{ A}$
- Ripple Current: $I_{AC} = 0.4 \text{ A}_{pk-pk}$ at 300 kHz
- Maximum Ambient Operating Temperature: $T_{max} = 100 \text{ }^\circ\text{C}$
- Maximum Component Temperature: $T_{max} = 125 \text{ }^\circ\text{C}$

The first step in designing an inductor is ideally to nail down the design requirements. In this example they are magically provided, but in reality knowing exactly what the inductor needs to handle is often a tough question.

This made up example is for a DC filter with a small, 20%, current ripple. The first question is: What type of core is needed? Considering the different material classes first is a reasonable approach. Saturation characteristics, permeability vs frequency, DC bias, and temperature are all parameters to consider.

- Ferrite Core:
 - Sharp Saturation, but the peak current is defined in this case
 - Usable at 300 kHz
 - High permeability, will need to be gapped for this application
 - Reluctance will be dominated by the air gap, so the inductance will be stable with temperature
 - Reluctance will be dominated by the air gap, so the inductance will be stable with DC bias
- Powder Core:
 - Soft Saturation, so it could likely tolerate higher peak currents
 - Usable at 300 kHz, although the higher μ cores will lose some permeability
 - Low permeability, no gap needed, and low permeability is appropriate for this application
 - Reluctance is dependent on material, but powder cores are very stable vs temperature
 - Soft saturation starts at low DC bias, so the core will lose significant permeability as DC current increases.
- Metal Core
 - Sharp saturation, but the peak current is defined in this case
 - **Not** usable at 300 kHz
 - High permeability, would need to be gapped

- Reluctance would be dominated by gap, so the inductance will be stable with temperature
- Holds permeability all the way up to saturation

This exercise limits our core types to gapped ferrite, or powder cores. Some interesting notes about the specific application requirements, only a minimum inductance was specified, and it was specified at a given DC current. While a ferrite core inductor will hold a steady inductance value all the way up to saturation, a powder core's inductance will vary dramatically with DC bias. For a filter inductor used at the input or output of a switching converter, typically the worst case noise occurs at maximum load, or maximum DC current. Additionally, the change in inductance may affect any filter resonance and damping. These effects are not always dealbreakers, but must be considered. In some applications where the peak current is not well defined, the soft saturation of a powder core may be a lifesaver.

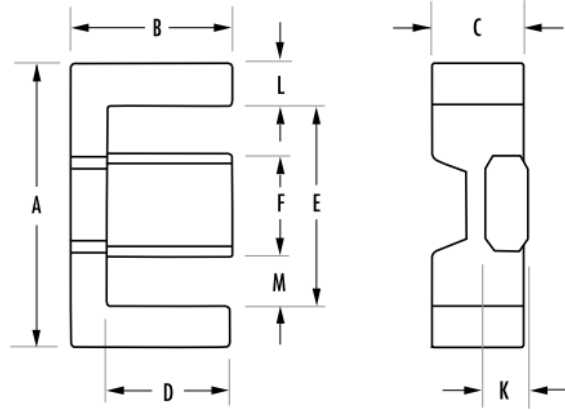
For discussion purposes, if the design requirements specified a minimum and maximum inductance, depending on the tolerance, the core type selection may shift towards ferrites. This is because the reluctance of an air gap is very stable with temperature and DC bias, so a gapped ferrite core will have a very stable inductance over temperature and DC bias, all the way up to saturation.

Practical assembly, availability, and mounting provisions may affect the choice as well. Small toroidal powder cores must be wound by hand, and may need to be potted for rugged mounting. Ferrites often are available with PCB mount bobbins that simplify winding and mounting, but will either need to be manually gapped, or custom ordered with the required air gap.

The only reason why metal cores are not discussed is because the most common materials are not usable at 300 kHz. Typically, metal core transformers are usable over the audio frequency range.

2.1.1 Ferrite DC Filter Inductor Analysis

Say ferrites are chosen as the path forward, and it's time to choose a specific core. Ferrite cores come in a wide variety of shapes. The following analysis applies to all shapes, but the EFD type ferrite cores will be used as an example. This specific shape has a low height profile, and every size listed has bobbins available to simplify winding and mounting. Solid toroidal ferrite cores are useless for power inductors because they cannot be manually gapped.



(a) EFD Core Half Drawing

		DIMENSIONS (mm)								
TYPE/SIZE	ORDERING CODE	A	B	C	D	E	F	K	L	M
EFD 10	0_41009EC	10.5 ± 0.3	5.2 ± 0.1	2.7 ± 0.1	3.75 ± 0.15	7.65 ± 0.25	4.55 ± 0.15	4.45 ± 0.05	1.43 ref	1.55 ref
EFD 12	0_41212EC	12.5 ± 0.3	6.2 ± 0.1	3.5 ± 0.1	4.55 ± 0.15	9.0 ± 0.25	5.4 ± 0.15	2.0 ± 0.1	1.75 ref	1.8 ref
EFD 15	0_41515EC	15.0 ± 0.4	7.5 ± 0.15	4.65 ± 0.15	5.5 ± 0.25	11.0 ± 0.35	5.3 ± 0.15	2.4 ± 0.1	2.0 nom	2.85 nom
EFD 20	0_42019EC	20.0 ± 0.55	10.0 ± 0.15	6.65 ± 0.15	7.7 ± 0.25	15.4 ± 0.5	8.9 ± 0.2	3.6 ± 0.15	2.3 ref	3.25 ref
EFD 25	0_42523EC	25.0 ± 0.66	12.5 ± 0.15	9.1 ± 0.2	9.05 min	18.1 min	11.4 ± 0.2	5.2 ± 0.15	3.15 ± 0.2	3.65 ± 0.2
EFD 30	0_43030EC	30.0 ± 0.8	15.0 ± 0.15	9.1 ± 0.2	11.2 ± 0.3	22.4 ± 0.75	14.6 ± 0.25	4.9 ± 0.15	3.8 ref	3.9 ref

(b) EFD Core Dimensions (EFD 10 K dimension is actually 1.45mm)

Table 2.1: EFD Core Magnetic and Window Data

Core	L_e	A_e	V_e	A_n	L_n
EFD 10	0.024 m	$7.2 \times 10^{-6} \text{ m}^2$	$171 \times 10^{-9} \text{ m}^3$	$5.56 \times 10^{-6} \text{ m}^2$	0.0196 m
EFD 12	0.029 m	$11.4 \times 10^{-6} \text{ m}^2$	$325 \times 10^{-9} \text{ m}^3$	$8.77 \times 10^{-6} \text{ m}^2$	0.0261 m
EFD 15	0.034 m	$15 \times 10^{-6} \text{ m}^2$	$510 \times 10^{-9} \text{ m}^3$	$13.3 \times 10^{-6} \text{ m}^2$	0.0359 m
EFD 20	0.047 m	$31 \times 10^{-6} \text{ m}^2$	$1.46 \times 10^{-6} \text{ m}^3$	$29.0 \times 10^{-6} \text{ m}^2$	0.0402 m
EFD 25	0.057 m	$58 \times 10^{-6} \text{ m}^2$	$3.3 \times 10^{-6} \text{ m}^3$	$41.4 \times 10^{-6} \text{ m}^2$	0.0500 m
EFD 30	0.068 m	$69 \times 10^{-6} \text{ m}^2$	$4.7 \times 10^{-6} \text{ m}^3$	$49.3 \times 10^{-6} \text{ m}^2$	0.0567 m

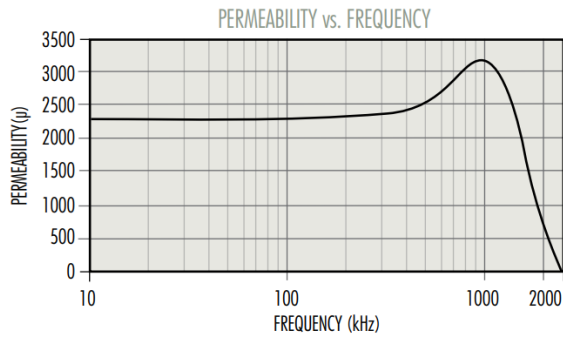
Ferrite Core Material Data

For power inductor design, a good first step is to calculate the gap required for each core. In this case, a precise minimum inductance is known, so we can use the formulas below to calculate the required reluctance and gap length.

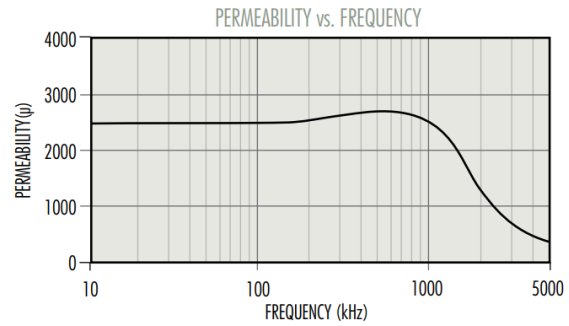
$$\mathcal{R} \geq \frac{L_{min} I_{peak}^2}{B_{sat}^2 A_e^2}$$

$$l_g = \mathcal{R} \mu_0 A_e$$

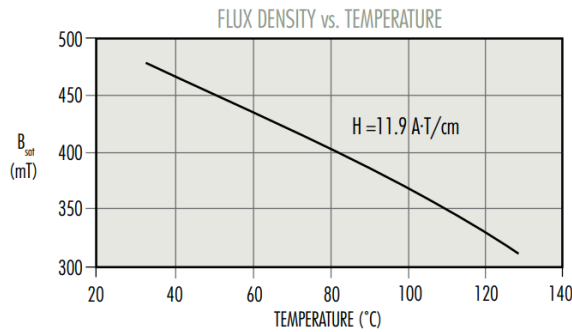
However, the ferrite material data is required to know what value should be used for B_{sat} . Most commonly available ferrites will saturate around 0.25 T to 0.4 T, depending on operating temperature. In general, most ferrites have a lower saturation flux density at elevated temperatures, and this application calls for operation up to 120 °C. This application has a low AC ripple current, so core losses will likely be negligible. Based on this, selecting a ferrite that has the highest saturation flux density that is also usable at 300 kHz will minimize inductor size and/or loss. Sometimes the material selection is limited by availability, and out of the engineers control. For this example, two commonly available materials from Magnetics Inc. will be compared: R and P.



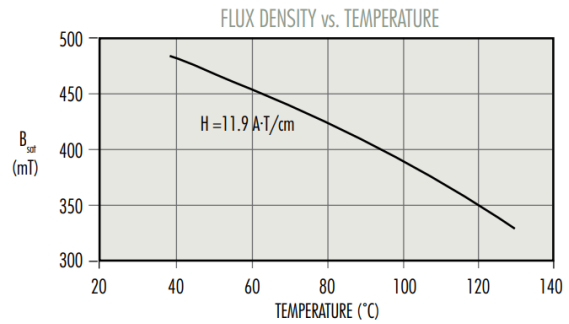
(c) R Material μ vs Frequency



(d) P Material μ vs Frequency

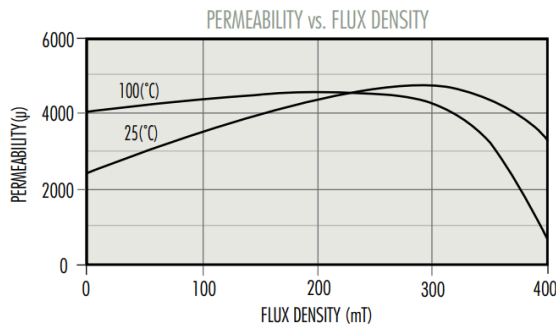


(e) R Material B_{sat} vs Temperature

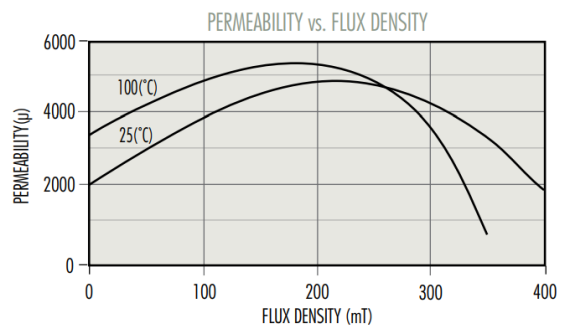


(f) P Material B_{sat} vs Temperature

From this material data, both materials are shown to be useful at 300 kHz. Since these cores will be gapped, the exact core permeability is unimportant since both are orders of magnitude larger than μ_0 . The curves also show that the P material has a higher saturation flux density for all temperatures. The application calls for operation at 120 °C, where the R material saturates at 0.33 T and P material saturates at 0.35 T. The B_{sat} vs Temperature curves show at what flux density the core will be completely saturated. These curves seem to show that the P material is likely a better choice, but in reality, usable inductance may begin to drop off at lower flux density than shown by the curves. To check this, a material permeability vs flux density curve is helpful.



(g) R Material μ vs Flux Density



(h) P Material μ vs Flux Density

The R material curve shows that at a flux density of 0.33 T and a temperature of 100 °C, the material still holds its permeability $\geq 2000\mu$. We can expect this curve to sink lower at 120 °C due to the downward slope of the permeability vs temperature curve, but there still seems to be margin. Remember, the total reluctance will be dominated by the air gap, so as long as the core keeps its permeability much higher than μ_0 , then it will be effective.

The P material curve shows that at 100 °C and 0.35 T the core permeability is approximately 750μ , and falling quickly. If the curve is worse at 120 °C, the core may not hold full inductance.

Required Gap, Reluctance, and Turns

Based on the material data, the R material appears to be the best choice for a conservative design, but both materials would be usable. A B_{sat} value of 0.32 T will be used for extra margin as well. Now a minimum reluctance and air gap length can be calculated for each core size. To simplify calculations, the core reluctance can be ignored, and the gap will be sized for the entire reluctance. Since this is an E type core, the air gap for a custom ground core will be ground into the center leg, resulting in a single air gap. The required turns will also be calculated.

$$\mathcal{R} \geq \frac{L_{min} I_{peak}^2}{B_{sat}^2 A_e^2}$$

$$l_g = \mathcal{R} \mu_0 A_e$$

$$N = \sqrt{L_{min} \mathcal{R}}$$

$$\mathcal{R} \geq \frac{(250 \times 10^{-6} \text{ H})(2.5 \text{ A})^2}{(0.32 \text{ T})^2 A_e^2}$$

Core	\mathcal{R}	l_g	N
EFD 10	394×10^6	$2.663 \times 10^{-3} \text{ m}$	271 Turns
EFD 12	117×10^6	$1.682 \times 10^{-3} \text{ m}$	171 Turns
EFD 15	67.8×10^6	$1.278 \times 10^{-3} \text{ m}$	130 Turns
EFD 20	15.9×10^6	$0.619 \times 10^{-3} \text{ m}$	63 Turns
EFD 25	4.54×10^6	$0.331 \times 10^{-3} \text{ m}$	34 Turns
EFD 30	3.20×10^6	$0.278 \times 10^{-3} \text{ m}$	28 Turns

After comparing the required gap lengths to the core cross section dimensions K and F, the EFD10 and EFD12 cores would likely have significant fringing flux. Since this application has a low AC ripple current, the fringing would likely not cause significant power dissipation, but the effective gap area would certainly be reduced, requiring a larger gap length than shown by the initial calculation.

Looking at the turns counts, keep in mind that for a given window size, $R_{DC} \propto N^2$, as increasing turns both increases the wire length and decreases the wire area. But in this table, the window area increased with core size, so $R_{DC} \propto N^4$ as shown on this table.

The results of this table are somewhat disappointing when it comes to core selection. All of the magnetics formulas presented so far are not enough to select a core and get on with the design. This is because at a fundamental level, the copper resistance and thermal properties of the design are the ultimate DC limiting factors.

DC Resistance

Analyzing the copper DC resistance is straightforward, but its accuracy is limited by practical factors. The basic method of calculating copper resistance is to divide the *usable* window area, A_n , by the number of turns required to calculate the conductor cross sectional area. Then the core or bobbin dimensions are used to calculate the average length of a single turn of wire, L_n , which is then multiplied by the number of turns to get the wire length. Then, the conductivity of copper is used to calculate the wire resistance.

$$A_{wire} = \frac{\eta A_n}{N}, \text{ m}^2$$

$$L_{wire} = L_n N, \text{ m}$$

$$R_{wire} = \rho_{Cu} \frac{L_{wire}}{A_{wire}}, \Omega$$

$$\eta = \text{winding packing efficiency}$$

$$\rho_{Cu} = 1.72 \times 10^{-8} \Omega \text{ m}^{-1} = \text{copper resistivity}$$

This basic formula is adequate for approximating copper DC resistance, but is even better for relative comparisons between core sizes. The two main sources of error come from the winding packing efficiency, and the fact that wire is only available in discrete sizes. The winding packing efficiency, η , can never be greater than 90.6% for round wire. That is the packing efficiency of a round wire in a perfect hexagonal pattern. The packing efficiency of round wire in a perfect square pattern is 78.5%. On top of that, all wires have added area from insulation. From experience, the insulation on HAPTZ magnet wire can add $\sim 77 \times 10^{-6} \text{ m}$ to the wire diameter. The area

consumed by wire insulation is very significant for small wires. Larger wires suffer from being hard to bend, and cannot be easily coaxed into perfect patterns, or perfectly tight coils.

For conservative initial design, a 50% packing efficiency is reasonable for a single winding. That value will be used to compare estimated DC resistances for all the EFD core sizes, and their loss at 2 A DC.

Table 2.2: EFD Core estimated DC resistance

Core	\mathcal{R}	l_g	N	R_{DC}	P_{loss}
EFD 10	394×10^6	2.663×10^{-3} m	271 Turns	8.913Ω	35.65 W
EFD 12	117×10^6	1.682×10^{-3} m	171 Turns	2.993Ω	11.97 W
EFD 15	67.8×10^6	1.278×10^{-3} m	130 Turns	1.565Ω	6.26 W
EFD 20	15.9×10^6	0.619×10^{-3} m	63 Turns	0.189Ω	0.76 W
EFD 25	4.54×10^6	0.331×10^{-3} m	34 Turns	0.048Ω	0.19 W
EFD 30	3.20×10^6	0.278×10^{-3} m	28 Turns	0.031Ω	0.12 W

These results show that the EFD 10 and EFD 12 core are severely undersized. The EFD 15 is approaching sane values, but is likely to overheat, but this can't be known until it is tested. These results also show that the jump from EFD 25 to EFD 30 provides marginal improvement for the size increase.

The best thing to do next is order prototyping parts for the EFD 15, EFD 20, and EFD 25 cores and work on something else until the parts arrive. For an application like this, with negligible AC losses to consider, no further analysis will allow an engineer to narrow down the core choice with significant confidence, without relying on previous empirical experience. Also, ordering multiple core sizes allows for adaptation to often changing design requirements. Depending on the application, ordering multiple core sizes of different shape families may be useful.

Automated and Analytical Core Selection

All of the formulas used to aid in core selection are simple and have closed form solutions. When starting a design, compile the data for several core sizes and use a spreadsheet to calculate reluctance, gap length, and DC resistance for the given design requirements. If all the core data and calculations are done in a single row using global design parameters, then all core sizes can be compared simultaneously.

2.1.2 Ferrite DC Filter Inductor Prototyping

Core Gapping

When parts do arrive, the practical challenges begin, and the hardest one is up first: gapping the core. Some manufacturers, like TDK/EPCOS, supply pre-gapped cores that are available from distributors like Mouser and DigiKey. With some luck, there may be a combination of gapped/ungapped, or gapped/gapped core halves that results in an acceptable gap length. If pre-gapped cores are not available, the next best option is to add thin layers of material between the two core halves. The gapping material is best placed on the outer legs of the core to ensure an even gap on both sides. This will also create a gap between the center legs of the core, so the required air gap length will be half the value calculated previously. For a DC inductor, any thin, nonmagnetic, and incompressible material can be used to set the gap. (Although for inductors with high AC flux, nonconductive materials should be used to avoid eddy current losses). A micrometer with 0.0001" or 0.01mm resolution can be helpful for measuring any available papers, tapes, and plastic films in an attempt to find an appropriate combination. For reference, standard printer paper is $\sim 0.10 \times 10^{-3}$ m thick.

To test the gap, a test winding and an LCR meter (or any other method of measuring inductance) is needed. The test winding can be the same as the number of turns used in the design, but generally is at least 50 turns to ensure an accurate measurement.

To assemble the core and test winding, place the gapping material on each outer leg of one core, add the bobbin, and the other core half. Secure the two cores together with several turns of tape. Small metal core clips/yokes can be purchased, but are not recommended. They are made of spring steel and lay flat against the air gap, changing their reluctance. With the assembled core, measure the inductance, and then calculate the actual reluctance.

$$\mathcal{R} = \frac{N^2}{L}$$

The calculated reluctance should be at least the value calculated earlier, or a few percent higher. A lower reluctance means the core would require fewer turns, but saturate earlier. A higher reluctance will increase turns, but would saturate at a higher current. Repeat the process until the best combination of gapping materials is found. Very thin materials like tissue paper (the stuff used for gift wrapping) or polyester electrical tape can help fine tune an air gap. plastic film tapes are especially useful because they are easy to stack and hold themselves in place.

Alternatively, if a milling machine or appropriate precision grinder is available, the gap can be ground into the center leg of an E core to introduce a single gap.

Wire Selection and Winding

With the actual reluctance measured, the actual coil must be wound on the bobbin. Wire size must also be calculated. Depending on how close the actual reluctance is to the design values, the number of turns may have to be adjusted.

$$N = \sqrt{L_{min} \mathcal{R}}$$

$$A_{wire} = \frac{\eta A_n}{N}$$

$$D_{wire} = 2\sqrt{\frac{A_{wire}}{\pi}}$$

$$AWG = 36 - 39 \log_{92} \left(\frac{D_{wire}}{0.127 \times 10^{-3} \text{ m}} \right)$$

Since the winding packing factor used to calculate A_{wire} was an estimate, and wires are available only in discrete sizes, try multiple wire sizes until the largest wire that easily fits with the required turns is found.

Then the prototype inductor can be fully assembled. Verify its inductance with an LCR meter. Don't be surprised if the LCR meter displays a ESR significantly higher than the expected DC resistance. The LCR meter isn't lying, that ESR is there, but it is an AC resistance. Measure the inductance and ESR at all test frequencies the LCR meter supports and note any trends. For a DC filter inductor, AC resistance can be ignored. But in other designs, AC resistance can pose a significant design challenge.

Temperature Rise

Luckily for a DC inductor, thermal testing is easy. Force the required DC current through it, measure it's voltage drop (with kelvin connections), and measure its temperature rise. Measuring the voltage drop is not required for thermal verification, but it is valuable. It provides the actual DC resistance, allows power loss to be calculated, and it can be used to calculate the core's effective thermal resistance to ambient temperature. Thermal resistance data is not readily available from manufacturers, and can be useful for future designs.

Saturation Current

The saturation current can be verified by using a power supply, a power MOSFET, a catch diode, a current probe (or current sense resistor), and a signal generator. Set the power supply to a voltage at least 10x higher than the expected $I_{sat} \cdot R_{DC}$ voltage drop. Monitor the inductor current with an oscilloscope, and set up the pulse generator to output short pulses with a very long period such that $t_{off} \cdot V_d \gg t_{on} \cdot V_{DC}$ so that the inductor current can decay to zero between pulses with only the diode forward voltage applied, and to keep power dissipation low. Start with sufficiently short pulses where $t_{on} \ll I_{sat} L / V_{DC}$. The upward slope of the inductor current should be a linear ramp starting from 0 A. Increase the pulse on time until the peaks of the current slope go nonlinear and the slope begins to increase quickly. This is the saturation point. As a side note, this test can be used to verify inductance if an LCR meter is unavailable, where $L = V_{DC} / (dI/dt)$.

This test should be performed over temperature. Cold testing may require a thermal chamber, but is often not as critical as hot testing. Hot testing can be done with self heating. With a sufficient power supply, the MOSFET and current sense resistor can be shorted, and the power supply current limit can be adjusted to heat up the inductor to maximum operating temperature. The short can be removed, and saturation pulse test can then quickly be performed to verify saturation current at elevated temperature.

If the saturation current of the inductor is significantly higher than what's required, the air gap could be made smaller, which would reduce the turns count, and DC resistance.

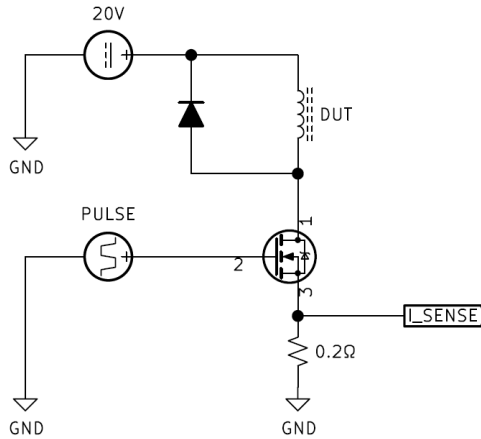


Figure 2.1: Inductor Saturation Current Test Setup

Impedance vs Frequency

The inductors impedance vs frequency should also be measured. A vector network analyzer is ideal for this purpose, but is not always available. Using a VNA to measure impedance magnitude and phase vs frequency will show all the resonant points. The most basic measurement would be of the inductors self-resonant frequency (SRF). This is the first point where the inductance resonates with parasitic winding capacitance, and also is the frequency where the inductors impedance is highest.

This can be measured with just a signal generator, resistor, and an oscilloscope. Pick a resistor that will have a higher impedance than the inductor up to ~ 10 MHz, so $R > 2\pi L \cdot 10$ MHz. Connect the signal generator to the inductor through the resistor and set it up for a sinewave output. Monitor the voltage across the inductor with an oscilloscope. At low frequency, the inductor will have much lower impedance than the resistor, so the voltage will be very small. Sweep the signal generator frequency until the maximum inductor voltage is found. This is the inductor's self resonant frequency. This frequency should be sufficiently above the noise frequencies it is designed to filter, so at least 3 MHz for this design example.

If the self resonant frequency is too low, there may be a few mitigation techniques. A low SRF is caused by high inter-winding capacitance. If the inductor was wound with multiple layers, winding one layer across the full width of the bobbin, then the next layer back on top of the first, etc., is the worst way to wind an inductor. It allows windings that have significant voltage differences between them to lie on top of each other, exaggerating the effects of their parasitic capacitance. If possible, wind partially across the bobbin, then back on top of the first partial layer, etc., building the winding height to the full window height before progressing down the width of the bobbin.

The next option would be to increase the core size. Fewer turns may provide a higher SRF. The larger core could also be used with a smaller wire, reducing the number of layers needed to achieve the required turns count. Fewer layers, combined with optimized layer winding will help increase SRF.

Ordering Custom Gapped Cores

Once the design is fully verified, custom gapped cores can be ordered. Companies like [MTL Distribution](#) offer small-run custom gapping services. The most accurate way to order a gapped core is typically by A_L value. The A_L value is simply the reciprocal of reluctance, so $A_L = 1/\mathcal{R}$, H/Turn². However the industry standard is to multiply that value by 10^9 and specify A_L in units nH/Turn². Gapped cores can be ordered as sets, and will be a combination of one ungapped core and a gapped core, or two gapped cores, depending on A_L value.

2.1.3 Powder Core DC Filter Inductor Analysis

Design of a powder core DC filter inductor is significantly different than designing a ferrite core inductor. While with a ferrite core the air gap provides a dominant and steady reluctance, with a powder core the material sets the reluctance, and has less than ideal properties. The key property of powder cores is their soft saturation. The overall effects of soft saturation is inductance will start to fall as DC current increases. This can be visualized by the common permeability vs \mathbf{H} field curves.

Permeability versus DC Bias Curves

MPP Toroids 14 μ - 200 μ

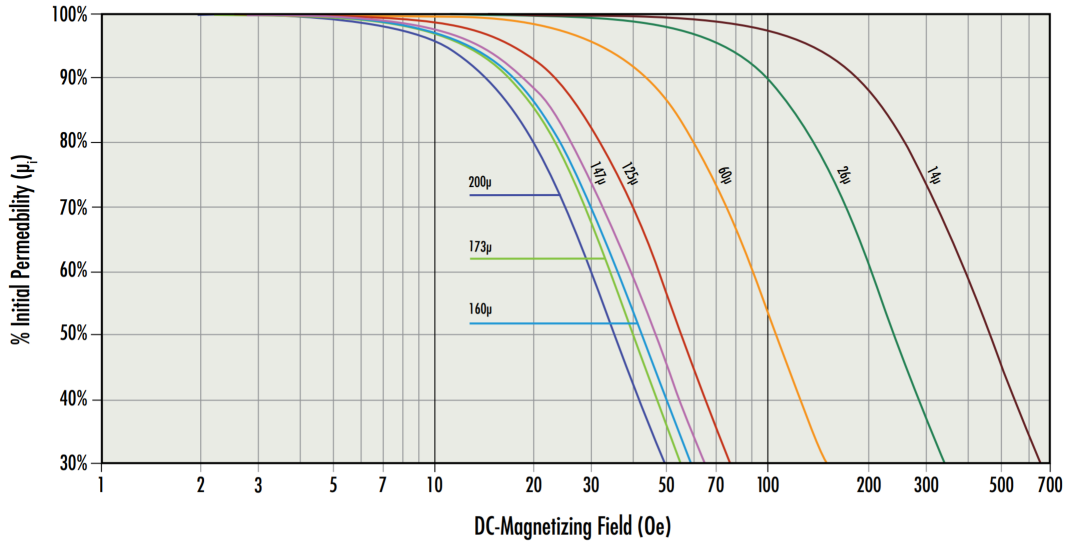


Figure 2.2: MPP μ vs \mathbf{H}

Looking at the 60 μ curve, it shows that when the \mathbf{H} field is just over 100 Oe, the permeability, and therefore inductance, will be 50% of the initial value. This variable permeability makes selecting the appropriate number of turns difficult, because as turns or increased to compensate for the lowered permeability, the magnetizing force is also increased. The [Magnetics Inc](#) powder core catalog has the curves, and curve fit formulas for all of their powder core materials, as well as visual selection guides for core sizes based on material and energy storage. However, thanks to Magnetics Inc being an American company that has only halfway modernized its core data, all the core and material data is a mix of SI and imperial units, with odd factors of 10^n sprinkled in. For use in any of the presented formulas all of it must be converted to SI units. Additionally, despite the curve fit formulas, due to their form, no simple closed form solutions exist for determining how many extra turns are required to compensate for soft saturation at a given DC bias.

The first step of selecting a powder core is to first choose the powder core material. For a DC filter inductor application, the critical properties will be high saturation flux density, and usable frequency range. Core loss can be ignored. Of course availability and cost factor in as well. A new consideration is how quickly the soft saturation sets in. Magnetics Inc. specifies the magnetizing force required for the material permeability to fall to 80% and 50% of its initial value, but only for the 60 μ version of each material. The higher the listed magnetizing force is, the more DC bias the material can tolerate while maintaining inductance. Higher permeability materials will lose permeability faster than the lower permeability options. The 60 μ permeability is used as an example because it is a common choice, and is approximately in the middle of the permeability range.

		Kool M μ [®]	Kool M μ [®] MAX	Kool M μ [®] Hf	XFlux [®]	High Flux	Edge [®]	MPP
Alloy Composition		FeSiAl	FeSiAl	FeSiAl	FeSi	FeNi	FeNi	FeNiMo
Available Permeabilities		14-125	14-60	26, 60	19-125	14-160	26, 60	14-550
Core Loss - 60 μ (mW/cc)	50 kHz, 1000 G	215	200	120*	575	250	150	165
	100 kHz, 1000 G	550	550	325*	1,280	625	375	450
Perm vs. DC Bias - 60 μ (Oe)	80% of μ_i	45	65	60	100	100	130*	60
	50% of μ_i	95	130	115	170	185	205*	105
60 μ Temperature Stability - Typical % shift from -60 to 200°C		6%	3%	5%	4%	4.5%	2%	2.5%
Curie Temperature		500°C	500°C	500°C	700°C	500°C	500°C	460°C
Saturation Flux Density (Tesla)		1.0	1.0	1.0	1.6	1.5	1.5	0.8
Frequency Response - 60 μ flat to ...		5 MHz	15 MHz	30 MHz	3 MHz	3 MHz	20 MHz	6 MHz
Relative Cost		1x*	2x	2x	1.2x	4x-6x	5x	7x-9x

*indicates best choice

Figure 2.3: Magnetics Inc. Powder Core Material Summary

From this data, the Edge, High Flux, and XFlux materials are the three best options. For this example, the High Flux material is chosen due to its availability in many toroid sizes.

Core Selection using Graphical Tools

Magnetics Inc. provides charts that aid in the selection of powder cores, once a material is selected. This design method is subject to what the engineers at Magnetics Inc. have decided is a good balance of size and power loss that will be generally applicable. Using these guides is a good way to get started, but of course, the final design will have to be adjusted based on testing to the specific design requirements. This method also makes it cumbersome to compare multiple core sizes side-by-side, since it does not lend itself to spreadsheet calculation.

The x-axis of the chart is truly a measure of energy storage, but uses odd units. The mHA^2 product for this design example is $0.250 \text{ mH} \cdot (2.0 \text{ A})^2 = 1.0 \text{ mHA}^2$. To use the chart, find 1.0 mHA^2 on the x-axis, follow that value up to the black line, and then follow the closest horizontal line to the left or right to see the recommended core. For this design example, the 160 μ 58118 core is recommended.

Core Selector Charts

High Flux Toroids

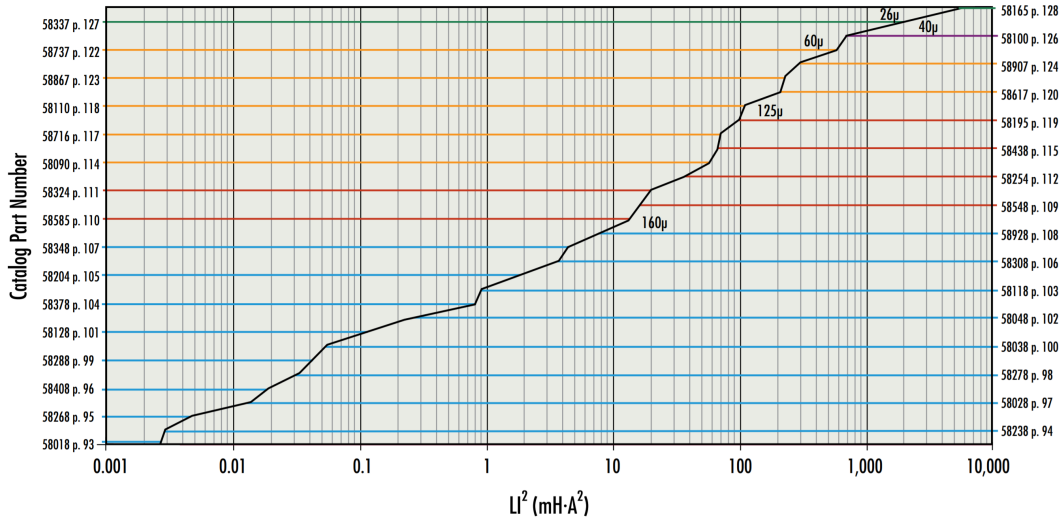
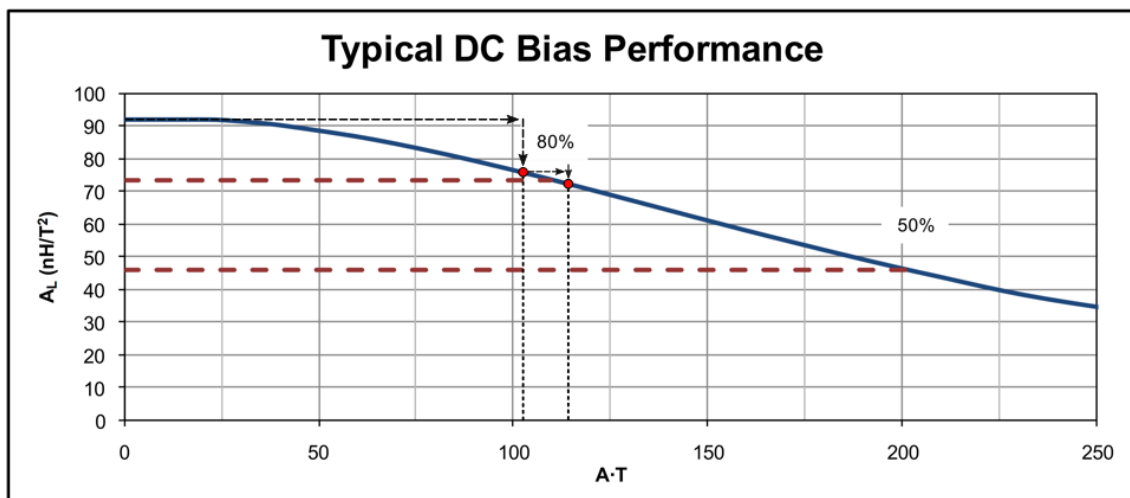


Figure 2.4: Magnetics Inc. High Flux Core Selection Guide

Selecting Turn Count Graphically

The datasheet for the core (or the powder core catalog) has another convenient graph that plots the core's A_L value vs the ampere-turn product. This graph is simply the material permeability vs \mathbf{H} field curve with the core dimensions factored in. The process for using this chart to find the number of turns required is iterative. First, start with the A_L value at no DC bias, and calculate the number of turns required $N = \sqrt{L/A_L}$. Then calculate the amp-turn product $A \cdot N = N \cdot I_{DC}$. The curve will show a new, lower, A_L value. Iterate this process with the new A_L value and the required turns will approach the real value. Usually 2 to 4 iterations is good enough, depending on the material.

Figure 2.5: Magnetics Inc. High Flux Core 58118 A_L vs $A \cdot T$ product

$$N_0 = \sqrt{\frac{250 \times 10^{-6} \text{ H}}{92 \text{ nH/T}^2}} = 52 \text{ Turns}, \quad 52 \text{ Turns} \cdot 2 \text{ A} = 104 \text{ A} \cdot \text{T}$$

$$N_1 = \sqrt{\frac{250 \times 10^{-6} \text{ H}}{76 \text{ nH/T}^2}} = 57 \text{ Turns}, \quad 57 \text{ Turns} \cdot 2 \text{ A} = 114 \text{ A} \cdot \text{T}$$

$$N = \sqrt{\frac{250 \times 10^{-6} \text{ H}}{71 \text{ nH/T}^2}} = 59 \text{ Turns}$$

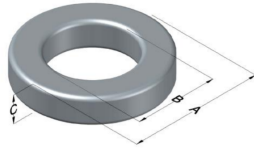
Calculating DC Resistance

With the number of turns, DC resistance can be estimated from the window area A_n (or W_A as shown on the datasheet) and the winding length per turn, L_n . Just like with ferrite core bobbins, the entire window area can't be utilized. The same 50% window packing factor will be used for estimating the DC resistance, and will be used to select the appropriate L_n value from the table on the datasheet.



C058118A2

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HK Sales: (852)3102-9337
magnetics@spang.com
www.mag-inc.com



High Flux Permeability (μ)	A_L (nH/T ²)	Core Marking			Coating Color
		Lot Number	Part Number	Inductance Grade	
160	92 ± 8%	XXXXXX	58118A2	X	Khaki

Dimensions	Uncoated		Coated Limits		Packaging
	(mm)	(in)	(mm)	(in)	
OD (A)	16.6	0.653	17.3	0.680	max
ID (B)	10.2	0.400	9.52	0.375	min
HT (C)	6.35	0.250	7.12	0.280	max

Bulk Pack
4 bags/box
Box Qty= 2000 pcs

Electrical Characteristics			Physical Characteristics						
Watt Loss @ 100 kHz, 100mT max (mW/cm ²)	DC Bias typical (oersteds)		Voltage Breakdown wire to wire min (V _{Ac})	Break Strength min (kg)	Window Area W _w (mm ²)	Cross Section A _c (mm ²)	Path Length L _e (mm)	Volume V _e (mm ³)	Weight (g)
2000	80%	50%	1000	23.0	71.2	19.2	41.2	791	6.4
	24.0	53.0							

Winding Information				Temperature Rating		
Winding Length Per Turn				Wound Coil Dimensions (mm)		
Winding Factor	(mm)	Winding Factor	(mm)	40% Winding Factor		Coating Temp (Continuous up to): 200°C
				OD	HT	
0%	22.1	40%	27.0	Max OD	23.7	Notes:
20%	24.6	45%	27.7	Max HT	15.2	
25%	25.2	50%	28.4	Surface Area (mm ²)		
30%	25.6	60%	29.8	Unwound Core		920
35%	26.4	70%	31.5	40% Winding Factor		1,300

Figure 2.6: Magnetics Inc. High Flux Core 58118 Datasheet

$$A_{\text{wire}} = \frac{\eta A_n}{N} = \frac{(0.5)(71.2 \times 10^{-6} \text{ m}^2)}{59 \text{ Turns}} = 603 \times 10^{-9} \text{ m}^2$$

$$L_{\text{wire}} = L_n N = (0.0284 \text{ m})(59 \text{ Turns}) = 1.68 \text{ m}$$

$$R_{\text{wire}} = \rho_{\text{Cu}} \frac{L_{\text{wire}}}{A_{\text{wire}}} = 1.72 \times 10^{-8} \Omega \text{ m}^{-1} \left(\frac{1.68 \text{ m}}{603 \times 10^{-9} \text{ m}^2} \right) = 0.048 \Omega$$

$$P_{\text{loss}} = 0.19 \text{ W}$$

This estimate of DC resistance shows similar performance to the EFD 25 ferrite core, but in a slightly smaller size. However, in the ferrite example, the inductor design would hold 250mH up to 2.5 A, where this powder core design will only hold 250mH up to 2.0 A. At this point the best next step is to order the 58118 core, and the next two closest sizes (58048 and 58378) and work on something else until the parts arrive.

Automated and Analytical Core Selection

Varying degrees of automation can be used to aid in powder core selection, all of which are based on the provided curve fit formulas. The only curve fit formula needed for a DC power inductor is the material permeability vs **H** field curve. The formula and coefficients for each material are provided in a table.

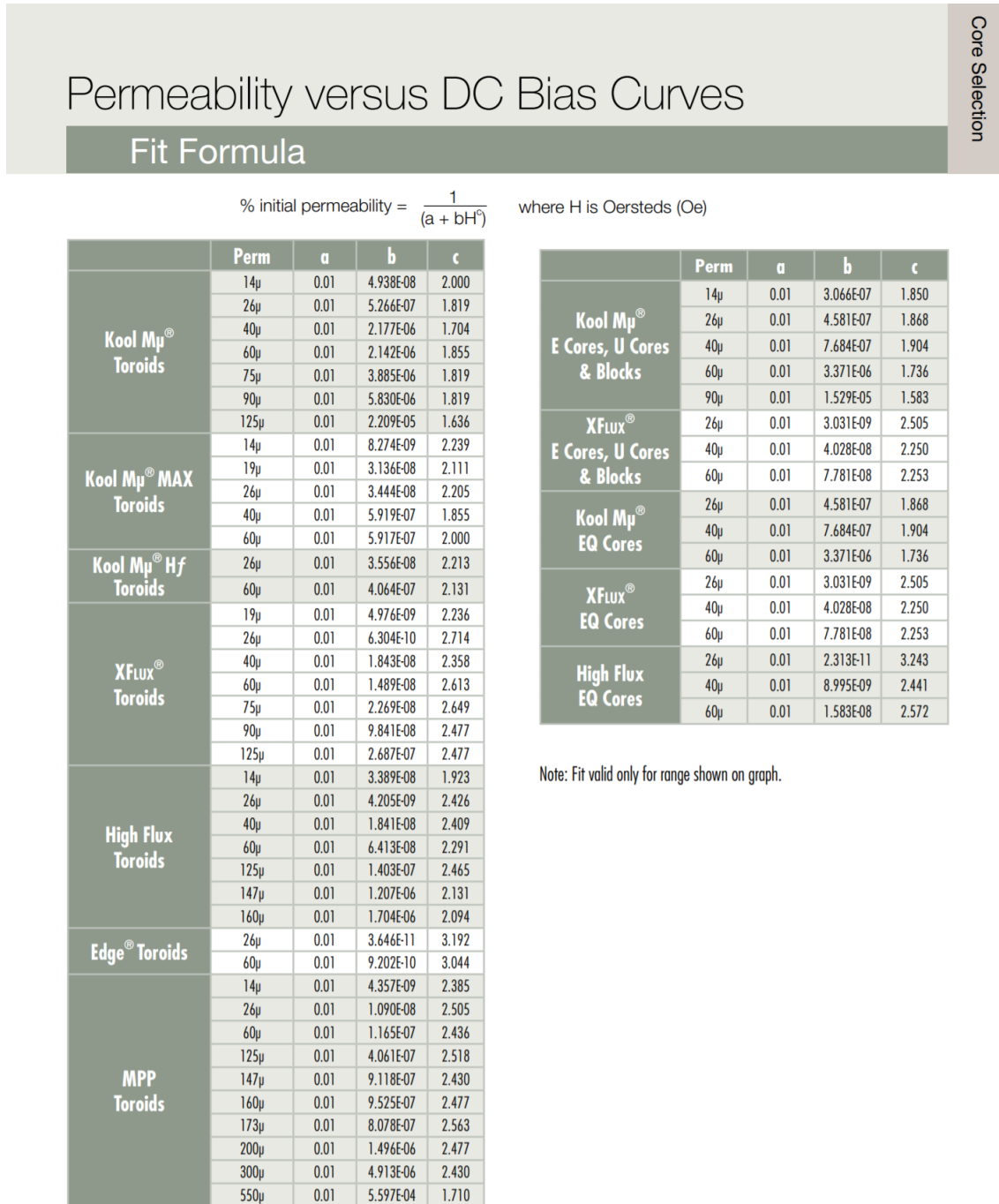


Figure 2.7: Magnetics Inc. Powder Core μ vs **H** curve fit coefficients

The first level of automation would be to use a spreadsheet where the user manually enters: Turns, I_{DC} , A_e , L_e , A_n , L_n , nominal material permeability (μ_i), and the curve fit coefficients from the table above. Those values would be obtained from the engineers selection of material, and the core selection charts used earlier. The A_L value and inductance at the given DC bias is calculated from the curve fit permeability and the core dimensions. The curve

fit formula is adjusted to accept SI units, and to output permeability relative to μ_0 (instead of percent of initial permeability).

$$\mu = \frac{\mu_i/100}{0.01 + b \left(\frac{1000NI_{DC}}{4\pi L_E} \right)^c}$$

$$A_L = \frac{L_e}{\mu\mu_0 A_e}$$

$$L = N^2 A_L$$

Since there is no simple closed form solution to solve for the correct turns count (unlike with ferrite cores), The turns count must be manually iterated until the desired inductance is shown. At the same time, the spreadsheet can calculate the wire resistance, loss, and AWG.

$$A_{wire} = \frac{\eta A_n}{N}, \text{ m}^2$$

$$L_{wire} = L_n N, \text{ m}$$

$$R_{wire} = \rho_{Cu} \frac{L_{wire}}{A_{wire}}, \Omega$$

$$P_{loss} = R_{wire} I_{DC}^2, \text{ W}$$

$$D_{wire} = 2\sqrt{\frac{A_{wire}}{\pi}}, \text{ m}$$

$$\text{AWG} = 36 - 39 \log_{92} \left(\frac{D_{wire}}{0.127 \times 10^{-3} \text{ m}} \right)$$

More advanced automation would involve compiling all the core data for the various toroid sizes, all the curve fit data for the various materials and permeabilities, and then automating the turns count iteration on a core-by-core basis. An excel macro, or python, could do this easily. A core size and turns count determined this way is no better than the graphical method, but it is faster and more adaptable in the long run. Although ultimately, the inductor design will have to be validated by testing.

2.1.4 Powder Core DC Inductor Prototyping

Construction of a prototype powder core inductor is simplified because no gapping is required, just wind the coil and start testing. However, winding toroids is always annoying. Thin wire bends easily, but often means the turns count is high, and it is impossible to wind a toroid quickly by hand. Thick wire is uncooperative, and due to its rigidity, it will take up much more space than expected.

Measuring Inductance

The main challenge with powder core inductor prototyping is actually measuring the inductance. An LCR meter is unable to provide the DC bias current, so the inductance measured will be significantly higher than the design value. The same test setup used to measure a [ferrite inductor's saturation](#) can be used to measure inductance over the whole DC bias range. While a ferrite core will exhibit a very linear current ramp when a voltage pulse is applied, the soft saturation on a powder core may cause the ramp to curve upwards. At any instantaneous current, the inductance is equal to $L = \frac{V}{dI(t)/dt}$. Measuring the $dI(t)/dt$ can be done with oscilloscope cursors. Setting the trigger to the current level of interest and zooming in on the slope around that point will give a more accurate measurement. Also, a high-resolution acquisition mode, and/or sample averaging may improve accuracy. If the signal is still too noisy, manually average several measurements, or export the trace data into excel for more filtering and measurement control. Selecting an appropriate DC voltage is also important. The DC voltage should be high enough so that all the resistive voltage drops from the inductor, MOSFET, and current sense resistor are negligible.

The same setup can be used to test the inductor up to the required peak currents. Although saturation will not be nearly as dramatic as with a ferrite core.

Thermal Testing

Temperature rise testing can be measured in the [same way as a ferrite core inductor](#). A DC power supply can be used to supply a DC current through the inductor, and the voltage drop, temperature rise, and power dissipation can all be measured. Again, record the thermal data, as no manufacturers provide good thermal data for toroidal cores. The inductance at DC bias should also be measured over temperature. While the initial permeability (and therefore inductance) of powder cores is very stable, there is no manufacturer provided data on how the permeability is affected by DC bias over temperature.

Impedance vs Frequency

The same concepts of SRF and impedance measurement that apply to [ferrite core inductors](#) also apply to toroidal inductors.

2.1.5 Metal Core DC Inductor Challenge

The initial design requirements ruled out metal cores for this example due to their poor performance at high frequency. But as an exercise, assume the design requirements have changed, and instead of a 300 kHz ripple frequency, the ripple is at 3 kHz, making a metal core a viable choice.

While EFD cores are not available in with metal core materials, for a more direct comparison, re-run the analysis for a hypothetical situation where all the EFD cores are available made from Supermendur, which has a B_{sat} of 2.0T. Calculate the required Reluctance and gap length required for each core size. Also calculate the expected wire resistance and power loss, but try and find the shortcut that *does not* involve calculating new wire areas and lengths.